Mathematics as Thinking

A Response to "Democracy and School Math"

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ABSTRACT

Math education in the United States remains resistant to systemic change, and our country pays the price. Stemhagen's article "Democracy and School Math" further confirms this trend. Despite repeated calls for reform, decades of research on how people learn, millions of dollars invested in teacher professional development, and years of politicized debate, the math wars rage on—between those who believe students have the capacity to construct their own mathematical ideas and others who insist mastery of the traditional canon must come first. Meanwhile, algebra failure among secondary students remains rampant and elementary education majors report the greatest rates of math anxiety of any college major. Adults and children alike joke about being terrible at math, seemingly unaware of the extent to which this innumeracy serves as a barrier to full participation in democracy as well as to the realization of their individual goals, hopes, and dreams. In the math education community itself, there is little discussion of the unique role mathematics can play in preparing students for democracy. In this short paper, I offer a more detailed conceptualization of democratic mathematics education and discuss the role of constructivism in bringing these ideas to fruition. I suggest that a shift in the power dynamic that characterizes most mathematics classrooms will be a key component in moving beyond the gridlock.

> *We believe the kind of systemic change necessary to prepare our young people for the demands of the 21st century requires young people to take the lead in changing it. (Moses & Cobb, 2001, p. 19)*

MATH EDUCATION IN THE UNITED STATES REMAINS RESISTANT to systemic change, and our country pays the price. Despite repeated calls for reform, decades of research on how people learn, millions of dollars invested in teacher professional development, and years of politicized debate, the math wars rage on—between those who believe students have the capacity to construct their own mathematical ideas and others who insist mastery of the traditional canon must come first. Meanwhile, algebra failure among secondary students remains rampant, and elementary-education majors report the greatest rates of math anxiety of any college degree. Adults and children alike joke about being terrible at math, seemingly unconcerned about how this innumeracy hinders full participation in our democracy or the realization of their individual goals, hopes, and dreams. In the math education community, few attend to the unique role mathematics can play in preparing students for democracy.

This article offers a more detailed conceptualization of democratic mathematics education and discusses the role constructivism might play in bringing these ideas to fruition. I raise concerns about the extent to which others, Stemhagen (2011) among them, seem to view constructivist pedagogy as a cure for the ills that plague mathematics education in the United States. While I agree that math educators who embrace a constructivist philosophy have made a critical step in becoming the teachers our students need them to be, I contend that moving beyond the gridlock will necessitate something more, namely a fundamental shift in the power dynamic that characterizes most K–12 mathematics classrooms as well as a reconceptualization of mathematics as a

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dynamic discipline to be explored and created rather than a static domain to be mastered without thought or question.

Stemhagen's Contribution

For those of us who have worked to improve K–12 mathematics education over the past quarter century, Stemhagen's findings (2011) offer compelling, empirical evidence for trends we too have witnessed among teachers trying to simultaneously meet professional standards (such as those published by the National Council of Teachers of Mathematics) and public mandates (like the testing and ratings associated with No Child Left Behind). While often perceived by the public as objective and neutral, mathematics teaching in the United States has become increasingly political work. At the heart of the struggle rest deeply held beliefs about the nature of mathematics as a discipline, its role in a democratic society, the capacity of students to master rigorous concepts, and the instructional strategies that best support student learning.

Critical questions remain unanswered for many:

- Why do so many students encounter failure in mathematics?
- What mathematics do all students really need as citizens of the 21st century?
- How can teachers support students not only in knowing more mathematics but also in having a greater affinity for the subject?
- What sorts of professional supports will best help teachers embrace and implement more innovative, inquiry-based, and reform-minded instructional practices in the mathematics classroom?

In an educational climate that increasingly values experimental design, research on teachers and teaching faces particular challenges due to the complexity of the setting, the work, and the many uncontrollable inputs that can affect the outcome of interest—a problem Stemhagen also encounters with his empirical model.

Working in this rather contentious and high-pressure context, mathematics teachers at all grade levels hold their individual beliefs about what mathematics is and is not (their own philosophy of mathematics), shaped largely by their unique experiences as both teachers and learners of mathematics. These beliefs have a direct impact on their instructional choices, particularly their implementation of innovative practices and the climate for mathematics learning that they cultivate in their classroom. Stemhagen's work confirms this.

However, as Stemhagen points out, seldom do teacher preparation programs, mathematics education courses, or professional development offerings provide experiences that explicitly focus on the philosophy of mathematics or teacher beliefs in this area. And in the face of high-stakes accountability pressures, teachers tend toward what they themselves have experienced as students, meaning pedagogy that is more transmittal or traditional in nature, even when their espoused beliefs would indicate otherwise (Weiss, Pasley, Smith, Banilower, & Heck, 2003). In addition, well-intended teachers generally over-report their use of more reform-minded practice—something to consider, given Stemhagen's exclusive reliance on teacher survey data. In prior studies of K–12 mathematics reform initiatives involving trained researchers who interview target teachers and observe their mathematics lessons, the data suggest that teachers generally see themselves as engaging in more constructivist or reform-minded practice than the researchers do (Banilower, Boyd, Pasley, & Weiss, 2006). This proves to be the case even when teachers create lessons using innovative, research-based instructional materials designed to support more progressive practice (St. John, Fuller, Houghton, Tambe, & Evans, 2005).

Among the challenges to any sort of instructional change in mathematics is the longstanding tradition of a particular lesson structure commonly found in American classrooms. Decades of research have documented a pattern that repeats itself regardless of time, place, or demographics (Welch, 1978; St. John et al. 2005). Most prevalent at the secondary level, but found regularly in elementary classrooms as well, it looks something like this:

1. Teacher begins the lesson with a warm-up or other launch activity.
2. Class corrects homework from the previous lesson.
3. Teacher presents new material.
4. Class practices new idea or technique.
5. Teacher assigns homework for the next class.

Repeat daily, week after week, year after year. It should come as no surprise that students become bored with their math classes and develop skewed perceptions of the discipline—as something static rather than dynamic, discovered rather than created, irrelevant to their lives, disconnected from human experience, and based on innate skill rather than hard work. Variations on the theme do occur. For example, students might present solutions to the homework or they might engage in an experiment as a way of exploring the concept at hand. However, by and large, the pattern and the power structure it reinforces remain in place, with the teacher as the sole source of knowledge and center of activity. Open inquiry and student-guided investigation are rare. Students do not own the mathematics; it owns them, leaving many students feeling defeated or less than worthy. For nearly half the population, insurmountable challenges in math serve as one of the most significant barriers to graduating from high school or finishing college. The pursuit of democratic mathematics education must include overcoming this culture of failure, something Stemhagen does not mention in his work.

What Do We Mean by Democratic Mathematics Education?

Over the past 30 years, many calls for K–12 mathematics reform have focused on the importance of making more advanced topics, particularly those associated with first-year and second-year algebra, accessible to all students as a means of promoting democracy (National Commission, 1983; National Council of Teachers of Mathematics, 2000; Moses & Cobb, 2001; Common Core State Standards Initiative, 2010). Most states now have high school graduation requirements that mandate two years of algebra study. While critics often view such policies as coercion and question the notion that a common curriculum for all students will produce educational equity (Noddings, 1997), the increasingly technological nature of our society suggests that certain math skills—particularly the capacity to generalize patterns, to solve unfamiliar problems, to make use of multiple strategies, and to articulate thinking—have become for the 21st century what reading and writing were for the 19th century (Moses & Cobb, 2001).

The focus on equalizing access and attainment for all students has skewed the dialogue in the field of mathematics education toward educating students for democracy, namely by giving students the skills they need to participate as citizens. Darling-Hammond (1996) makes a critical distinction between education for democracy and education as democracy. In the latter, students have the opportunity to learn as part of a community in which they have a voice and can participate in making decisions with one another, leading to an authentic understanding of multiple perspectives. Still, as Ball, Goffney, & Bass (2005) discuss, even the most talented elementary teachers, who cultivate rich conversations in their classrooms whereby students embrace different perspectives and demonstrated sensitivity to different cultures, find difficulty doing the same when teaching mathematics. More than any other subject, mathematics lends itself to competition, testing, levels of attainment, and therefore, feelings of supremacy as well as inadequacy. This is especially the case when the purpose of mathematics education becomes further mathematical growth, rather than the development of students as competent and contributing citizens (Noddings, 1993).

In nearly every arena, from personal finance to health care, navigating life in the 21st century requires mathematical thinking, particularly problem solving. Society is increasingly complex and global in nature. Bombarded with information at every turn, we process more in a matter of minutes than many of even our recent ancestors did in their entire lifetime. Never has survival depended so much on the ability to reason logically, to discern facts from fiction, and to make judicious decisions based on the available data. Even careers historically considered low-skill require workers to estimate, to recognize patterns, to reason proportionally, and to use computerized tools that take commands in the form of mathematical statements. All of these situations not only require citizens to think critically but also to communicate their ideas—and here mathematics provides a seldom-recognized opportunity.

Student Agency

While Americans may not associate heated debate and the rich exchange of ideas with their math classes, truth be told, mathematics affords opportunities for carefully reasoned, thoughtfully articulated, fact-based classroom discourse that other subjects do not. Cultivating a community of learners that includes the teacher as one of those learners, thereby disrupting the traditional patterns of power and authority in the mathematics classroom, is key. Under such conditions, students work individually and collectively to generate a range of interpretations and representations of a single problem. Students learn to justify their reasoning and defend their positions with ideas. Disagreements can be solved through carefully constructed arguments rather than loud shouting or emotional manipulation (Ball et al., 2005)—in this context, mathematics educators make the shift from mathematics teaching and learning for democracy to mathematics teaching and learning as democracy.

Stemhagen touches on some of these ideas early in his article, citing how a range of scholars have viewed "the mathematics education–democracy link" (p. 2): He cites Gutstein's notion of "school mathematics as tools to help understand and analyze social inequities"; Moses and Cobb for acknowledging "ways school mathematics serves as a gatekeeper, allowing those who do well to move on to college and relegating those who do not to a noncollege track"; and Boaler, Curry, Moses and Cobb, and Villalobos for their work "to undermine the differential levels of attainment according to race, gender, or other category of marginalization" (p. 2). The author suggests that missing from prior scholarship is the notion of agency and the extent to which students can "use their mathematical knowledge and skills to solve problems germane to their lives and even to make the world a better place" (p. 2). While I wholeheartedly agree with the importance of student agency in the conceptualization of democratic mathematics education, I believe Stemhagen misrepresents the scholarship of the researchers cited by suggesting they do not attend to this critical factor in their work.

Gutstein (2006), whom Stemhagen references with regards to the first point of research, clearly states the importance of students using mathematics to make sense of and to change unjust situations: "Students can understand their own power as active citizens in building a democratic society and becoming equipped to play a more active role in that society" (Gutstein & Peterson, 2006, p. 2). When describing the radical work of the Algebra Project, in which high school and college students essentially become math activists, Moses & Cobb explain how "young people finding their voice instead of being spoken for is a crucial part of the process" (2001, p. 19)—which doesn't quite fit with where Stemhagen places their scholarship, as a second explanation for the mathematics-democracy link. Similarly, relative to Stemhagen's third research citation, Boaler suggests, "When school students are given opportunities to ask their own questions and to extend problems into new directions, they know mathematics is still alive, not something that has already been decided and just needs to be memorized" (2008, pp. 27-28).

Contrary to Stemhagen's indications, all of these researchers fully understand the importance of agency in the process of democratizing mathematics teaching and learning. They recognize that democratic mathematics education occurs when students do math, rather than have math done to them. They have spent their careers studying strategies for making mathematics a subject that builds students up rather than one that brings them down, and for supporting teachers in improving their practice. They understand that supporting students in developing their mathematical voice plays a pivotal role in this process.

The Role of Constructivism

> *If children are taught mathematics well, it will teach them much of the freedom, skills, and of course the disciplines of expression, dissent, and tolerance, that democracy needs to succeed. (Hannaford, 1998, p. 186)*

Democratic mathematics education can not take place in a classroom where the teacher insists students learn the same way, work toward a single best solution, minimize interaction and teamwork, and focus on the mathematical ends or answers rather than the means or processes. According to the constructivist approach, students should be given the opportunity to discover and construct their own mathematical meanings. It follows that teachers should play the role of facilitator, refraining from providing direct instruction or telling students anything that they might be able to discover for themselves.

Stemhagen devotes considerable attention to teachers' enactment of constructivist versus transmittal pedagogical practices, noting that even when teachers espouse a constructivist view of mathematics, they find constructivist practices more difficult to implement. This phenomenon has played out in other studies in which teachers explain the challenges they encounter when trying to implement reform-minded teaching strategies:

- They did not learn math themselves using these strategies.
- They have not seen strong models for using these strategies in real classrooms.
- They feel a lack of confidence in and control over the lesson.
- They fear the questions that students may ask if the math becomes too open-ended.
- They have trouble tolerating the noise associated with constructivist instructional practice.
- They question the effectiveness of small groups for engaging all learners (Banilower et al., 2006; St. John et al., 2005).

Stemhagen's discussion of the issues inherent in constructivist pedagogy and their connection to democratic mathematics education would benefit from a more thorough discussion. As Noddings explains, "Constructivism as a pedagogical orientation has to be embedded in an ethical or political framework" (Noddings, 1993, p. 159). While Stemhagen articulates the supportive connection between constructivist teaching and ideas for K–12 mathematics reform articulated in the *Principles and Standards for School Mathematics* published by the National Council of Teachers of Mathematics (2000), he seems to suggest that constructivism leads consistently and unquestionably to positive outcomes for students. However, some research indicates that constructivist practice can have unintended consequences. As with any pedagogical practice, implementation rests with the individual teacher, which leads to variation. In this case, teachers make different choices when it comes to: interpreting and enacting the facilitator role, grouping students, asking questions, and selecting tasks for student exploration. As a result, the development of key skills and ideas can be left somewhat to chance, and under these circumstances, some students fare better than others (Ball et al., 2005). Moreover, different cultures have different norms about hierarchy and classroom discourse. Individual students also have strengths and challenges they bring to the classroom. Therefore, if the teacher does not make use of a variety of strategies to meet the range of students needs, constructivist practices can perpetuate bias and exacerbate problems of inequity rather than reduce them.

As a result, even though constructivist teacher beliefs and classroom practices may play a significant role in cultivating mathematics education both for democracy and as democracy, the shifts that need to take place in the teaching and learning of mathematics for the 21st century reach beyond constructivism. They rest with teachers', students', parents', and administrators' deeply held beliefs—about what mathematics is, who should learn it, and why. They also involve a revisioning of the role of the student in the mathematics classroom—a process that may begin with constructivism but extends to political activism. For example, in their book *Rethinking Mathematics: Social Justice by the Numbers*, editors Gutstein and Peterson included numerous examples of students investigating local problems such as overcrowding in their schools or the distribution of public services in their neighborhoods (2006). In *Radical Equations*, Moses and Cobb take this one step further: "One of the crucial issues in my mind is whether this generation of young people will begin to demand the literacy in mathematics it is assumed they do not want, demand this tool that is so essential for meaningful citizenship today" (2001, p. 171). In short, our students need to become critical consumers of mathematics teaching and learning who can advocate for what they need, namely to experience mathematics as thinking.

A Model for Democratic Mathematics Education—Mathematics as Thinking

> *The origin of thinking is some perplexity, confusion, or doubt. Thinking is not a case of spontaneous combustion; it does not occur on "general principles." There is something specific which occasions and evokes it. (Dewey, 1910, p. 12)*

What if students did not associate mathematics with memorizing techniques out of context, completing long lists of mindless exercises, and being judged on the number of right answers? What if, instead, they viewed mathematics as the subject that made them think deeply while seeking the answers to compelling questions, many that they generated themselves? What if math class were something that students eagerly anticipated because they were excited about the challenge of exploring problems and issues that they value as important and relevant? This is mathematics as thinking. It occurs in classrooms where courageous teachers fully embrace the opportunity to learn with and from their students. In such classrooms, students reason independently to design their own problems and investigations, to verify their methods, and to justify their conclusions. The active learning that takes places goes beyond hands-on; it is minds-on.

Ellis and Malloy (2007) propose a framework for democratic mathematics classrooms that extends beyond constructivism and moves in the direction of mathematics as thinking. They suggest four key elements to define such classrooms: (a) problem-solving curriculum, (b) culture of inclusiveness and rights, (c) equal participation in decisions, (d) equal encouragement for success. In such classrooms, students work collaboratively to solve problems that they value as important in their lives. Building on their prior and collective knowledge, students develop the skills to locate relevant information when they need it and to use multiple representations to gain new insights. They view their math class as a forum for open discussions about a range of ideas and experience instructional strategies that affirm the value of diverse methods. In the democratic mathematics classroom, all students have the resources and support they need to actively engage with the task at hand and to develop the mathematical habits of mind needed to critically evaluate data for social action (Ellis & Malloy, 2007; Boaler, 2008).

The role of student voice is implicit in the model articulated above. Students must have the opportunity to regularly communicate their ideas through writing and speech; they must feel they have a say in their own learning and in the direction of classroom discourse. However, rarely does this occur in real classrooms. As evidenced in a frequently cited study, "teachers far outtalk *all* of their students together during 150 minutes of daily talk recorded in hundreds of classrooms . . . with student-initiated talk consuming only 7 or 8 minutes on the whole [out of 150 total]" (Cohen & Lotan, 1997, p. ix). Math is not a spectator sport. In order to think mathematically, students need to do mathematics, actively and vocally. This necessitates solving problems, real problems.

Real problems are not necessarily word problems, although they can be. A real problem is a question to which the answer is not immediately apparent. Too often in mathematics classes, students work with exercises rather than with problems (Herr, Johnson, & Piraro, 2001). Exercises, as the term suggests, provide opportunities for students to practice a particular skill. Students generally look at the question and know immediately how to arrive at an answer. A real problem (and therefore, a good problem) places students in a more challenging (and compelling) situation, namely needing to determine what to do in the face of not knowing what to do. However, due to their prior experiences in mathematics, most students struggle with this kind of ambiguity.

Too often, curriculum guides equate problem solving with solving word problems. Such problems generally appear at the end of chapters in math textbooks. Students must generate an equation or exercise to represent a written description of a context and they often struggle with these word problems, seeing them as neither interesting nor relevant to their life experience. It is this pattern in mathematics education that perhaps most threatens the democratic process. Currently, nearly 50% of high school students in America's urban areas drop out prior to graduation (Dillon, 2009). In a recent study of student engagement, the majority of students cited boredom as the reason for leaving school. Respondents defined boring as "material wasn't interesting" (74%) and "material wasn't relevant to me" (39%) (Willis, 2010). Not surprisingly, studies of math's role, particularly algebra, in the dropout problem yield similar results.

Instead, students need to experience authentic problem solving as George Polya wrote about more than half a century ago his famous work, *How to Solve It* (1945). For Polya, arguably one of the most influential mathematicians of the 20th century, mathematics *is* problem solving, and he articulated a general four-step process for solving any problem: (a) understand the problem, (b) devise a strategy, (c) carry out the plan, and (d) look back on your results—then revise and repeat if necessary.

In order to keep the problem interesting and to avoid getting stuck, Polya suggested we continue to examine the problem from a variety perspectives:

> *Trying to find the solution, we may repeatedly change our point of view, our way of looking at the problem. We have to shift our position again and again. Our conception of the problem is likely to be rather incomplete when we start the work; our outlook is different when we have made some progress; it is again different when we have almost obtained the solution. (Polya, 1945, p. 6)*

According to Polya, the use of diverse methods and representations, generated by the problem solver and applied to unfamiliar situations, rests at the heart of developing mathematical knowledge for students and teachers alike: "What is know-how in mathematics? The ability to solve problems—not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity" (1962, p. viii). Polya's vision, which defined his own work with preservice teachers and has influenced mathematics education reform for more than 50 years, remains at the heart of any efforts to democratize American mathematics classrooms.

Supporting the Shift

> *Education organized around a reasonable number of broad talents and interests, augmented and filled out by serious inquiry into common human problems, stands the best chance of achieving a meaningful equality. Such education, in which students are active co-creators of curriculum, is a truly liberal education for both personal and public life in a democracy. (Noddings, 1997, p. 29)*

The democratic mathematics classroom results directly from the vigilant work of a committed teacher who believes in the capacity of all people to learn mathematics, who creates space for the stories and voice of each student, who honors the expertise of every child in the room, and who promotes alternative points of view, multiple strategies, and divergent solutions. Such teachers not only love mathematics but also possess a rich toolkit of strategies with which to connect the mathematics at hand with interests and cultures of their students. They have managed to navigate the tension between content and pedagogy that creates such a conundrum for so many of their colleagues. In such classrooms, mathematics may indeed serve as a vehicle for fostering understanding among students of differing backgrounds and worldviews.

Teacher Preparation and Professional Development

Stemhagen's work, along with that of those cited here, suggests that the systemic effort to create a more democratic mathematics classroom begins with teacher preparation and the investment in ongoing professional development for mathematics teachers at every grade level. In order to democratize their practice, teachers must have mathematical experiences that shape their beliefs—about mathematics, about their own capacities as mathematical thinkers, and about their students—in a deeply personal way.

Stemhagen provides only limited demographic information about his sample—for example, 62% of the participants teach grades four or five and 37% teach in grades six through eight. Only 10% of the respondents indicate that their highest degree is in mathematics. The author reports no numbers with respect to SES, gender, or ethnicity. Therefore, it is fair to assume that the sample represents the current American teaching population, which is disproportionately White, female, and middle class (Ball et al., 2005)—and quite possibly, math anxious. Prior studies indicate that 90% of elementary school teachers in the United States are women and elementary education majors report the highest rate of mathematics anxiety of any college major (Beilock, et al, 2010; Hembree, 1990). Many among this population hold memories of less-than-positive childhood experiences with mathematics that contribute to their anxiety about teaching math to others. The current climate of raised mathematical standards (Common Core State Standards Initiative, 2010), whereby all teachers in grades four through eight play a pivotal role in preparing students for algebra, further exacerbates such issues. If teachers have a tentative relationship with mathematics, then they will tend to hold the mathematics close rather than turning it over to student exploration and inquiry (Bursal & Paznokas, 2006; Kahle, 2008).

However, new and veteran teachers find benefit in articulating their own experiences as learners of mathematics and seeing how these recollections, often quite painful, can shape their beliefs and practices. Below, a student teacher working in a seventh-grade classroom reflects on his own memories of algebra, a subject he is now responsible for teaching:

> *Algebra was the beginning of the end for me, I guess. It was the first big taste of math that seemed useless. It was vague and hard to understand and seemed to serve no real purpose. Algebra seemed like a lot of formulas to memorize, and it was even harder for me than the math that preceded it.*

The recounting of such stories serves as a pivotal first step in empowering teachers to create a vision of mathematics teaching and learning in their own classrooms. Developing a sense of mathematical efficacy through problem solving is a critical and necessary next step, one that requires sustained effort throughout the teaching career. Unless teachers experience and believe in their own mathematical abilities, they will find it difficult to cultivate the same in their students. Below, a new teacher shares her thoughts on beginning this process through a course on algebra for elementary teachers:

> *I generally don't like math, but I felt so comfortable in our doing math together, that ultimately the course ended up feeling like a treat. This was probably the first math class I took where I didn't feel stupid. I know that's probably not saying a lot because we were doing pretty basic stuff, but I never felt the panic that math and math classes usually cause. I felt like I learned a lot. Most of all, I learned to trust my own ideas and I enjoyed doing it. Math was exciting! This is what I want for my students.*

While only at the beginning of her career, this new teacher has had the opportunity to face her fears and redefine her relationship with mathematics in hopes of better supporting the learning of her students. Such experiences offer a way to counter what recent research indicates is highly likely to occur if teachers' math anxieties go unchecked, namely that they will inadvertently project their anxieties onto their students, girls especially (Beilock, 2011).

Moving Forward

So where do we go from here? Stemhagen's work, combined with the current discussion, suggests mathematics teacher educators and professional developers must attend very carefully to teacher beliefs and philosophy, particularly their relationship to instructional practice. While we may endorse constructivism as a supportive stance for the changes in mathematics teaching and learning we seek, we must also acknowledge the potential problems that can play out in classrooms, especially as our schools serve an increasingly diverse student population. In addition, we must do all that we can to support the development of teacher-student relationships, whose foundation is mutual learning rather than hierarchy. We can model this by creating opportunities for teaching candidates to engage in mathematical exploration and problem solving, whereby we get the chance to learn from their insights and they experience the power of learning from each other. Finally, we must find ways for K–12 students to become critical consumers of mathematics education, to learn to advocate for themselves and what they need to learn mathematics successfully.

When it comes to promoting democracy in the mathematics classroom of the 21st century, the work is systemic. It involves multiple constituents (teachers, students, administrators, and parents) having experiences that lead to foundational beliefs, such as:

- All students are capable of learning powerful mathematics.
- Mathematics was invented by humans, and we are still inventing it.
- Students can and should help design their mathematical learning experiences.
- Thinking mathematically means solving problems to which we do not know the answer.
- Successful education requires meaningful relationships between students and teachers.
- Algebra, like any other area of mathematics, is a way of thinking, not a class to pass; students can and should begin developing their algebraic thinking in elementary school.

We live in a time that focuses entirely too much attention on testing and standards, at the expense of thinking and competence. The ideas expressed above are not commonly shared among all teachers of mathematics. Gaining more widespread support will mean asking hard questions like, "Who is doing the work in this math lesson?" If the answer is "the students," then we are heading in the right direction. If the answer is "the teacher," then what can be done to navigate the balance between teacher authority and student voice in the mathematics classroom? Such a shift seems less about implementing constructivism and more about negotiating power. Just as American democracy is of the people, by the people, and for the people, promoting democratic mathematics education means fostering a classroom culture that is increasingly of the students, by the students, and for the students.

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