Democracy and School Math:
Teacher Belief-Practice Tensions and the Problem of Empirical Research on Educational Aims

Kurt Stemhagen

Abstract
This article describes an empirical project that studied fourth- through- eighth- grade math teachers' beliefs about teaching and learning and about the role of teaching and learning in broader society. Specifically, it examined relationships between teachers' reported beliefs and their use of transmittal, constructivist, and democratic classroom practices. The article concludes with consideration about the difficulties inherent in attempting to use empirical research to study our broad educational aims, particularly our democratic ones.

This article provides an account of an empirical project that studied fourth- through- eighth- grade math teachers' beliefs about teaching and learning and about the role of teaching and learning in broader society. Specifically, this study examined relationships between teachers' reported beliefs and their use of transmittal, constructivist, and democratic classroom practices. The article concludes with consideration about the difficulties inherent in attempting to use empirical research to study our broad educational aims, particularly our democratic ones.

This project was carried out under the auspices of Metropolitan Educational Research Consortium (MERC), a partnership between a particular school of education and seven school districts. MERC, in its own words, "provides a structure and process for conducting and disseminating applied research that will improve our educational system and enhance student learning" (2011). It can be difficult to get a project approved, particularly if it is not directly and immediately designed to address the districts’ most pressing needs and concerns (one of which is increased achievement via standardized tests). Buy-in must be fostered by the project's study team (in this case, a group of twelve made up of math teachers, math coaches, and district-level math specialists) and the policy and planning council (primarily superintendents, district-level directors of research, and school board members). While MERC is most certainly a space where university professors’ and school districts’ interests come together, it seems that, often, school districts’ desires are foregrounded.

As Leatham (2008) pointed out, earlier research in the area of math teacher beliefs and practices often attempted to study links between particular beliefs and practices. The conceptual model operational in this study involves looking at teachers’ beliefs as existing in a constellation, or a web, and considering how this whole set of beliefs relates to teacher practices (see Figure 3). Of particular interest is how teachers conceptualize their role as math teachers with regard to the democratic aims of public schooling. In other words, do math teachers view themselves as democratic educators in the sense of helping to foster the growth of civic-mindedness in their classrooms? If so, how do they see themselves as teaching math in a way that is consonant with these broad democratic aims? If not, what do they see as the purpose of school mathematics?

In order to test this web model of belief, this study looks specifically at teachers’ beliefs regarding the nature of mathematics.

Kurt Stemhagen is associate professor in Virginia Commonwealth University’s School of Education.

Acknowledgements: I am indebted to Jason Smith, the graduate assistant who helped with this study. I also acknowledge the Metropolitan Educational Research Consortium (MERC), particularly the Study Team and the Policy and Planning Council. Finally, thanks to Jim McMillan, MERC director, and Gwen Hipp, MERC managing director, for all of the support and guidance.
the nature of teaching and learning, and their teaching efficacy. The following literature review is offered as a sketch of how each construct is conceptualized. Also included is a brief review of relevant previous research in related areas.

**Literature Review**

**CONCEPTUALIZING MATHEMATICS AND MATH EDUCATION**

There are a variety of ways to think about mathematics education (the nature of its subject matter; its purposes, content, and methods). According to Ernest (1989), teachers’ views are a “system of beliefs” (p. 20) that often serves as a de facto philosophy of mathematics. It is important to note that teachers often do not articulate a fully formed philosophy of mathematics (Thompson, 1992) and, indeed: “teachers’ conceptions of the nature of mathematics by no means have to be consciously held views; rather they may be implicitly held philosophies” (Ernest, 1989, p. 20). That these ideas matter is well documented in the literature (Thom, 1973; Lerman, 1983; Thompson, 1985; Ernest, 1987; Pajares, 1992; Schoenfeld, 2001). Hanadl and Herrington (2003) effectively summed up this vein of research: “A growing body of literature shows that mathematics teachers’ beliefs affect their classroom practices although the nature of the relationship is highly complex and dialectical” (p. 59).

There are many ways to categorize various philosophies of mathematics and, hence, a variety of ways teachers might think about mathematics. One fairly typical way is to characterize philosophies of mathematics as falling into two main categories: mathematical absolutism and mathematical constructivism. This split, although sometimes occurring with slightly different terminology, is widely recognized (Hersh, 1997; Kitcher, 1983; Kline, 1980; Stemhagen, 2004). Absolutism is a way of thinking of mathematics as certain, permanent, and independent of human activity. Constructivism, on the other hand, centers around the ways in which humans actually create mathematical understanding and knowledge (Stemhagen, 2009). These philosophies do not necessarily manifest themselves in particular ways with regard to teacher practices, although it seems reasonable that there is a certain selective affinity between a constructivist philosophy of mathematics and constructivist teaching methods on the one hand and an absolutist philosophy of mathematics and more traditional teaching methods on the other hand. The findings and discussion sections below address this issue in some detail. One contribution of this particular study is that it seeks to go beyond the constructivist/absolutist dichotomy in philosophy of mathematics and also beyond the constructivist/traditionalist dichotomy in the teaching of mathematics. The introduction of the idea of democratic mathematics education is intended to trouble these simple either-or settings by adding a layer of complexity, how teacher intent relates to practice.

**DEMOCRATIC MATHEMATICS EDUCATION**

Public schooling has civic aims that are often strangely absent in mathematics class (Stemhagen & Smith, 2008). Democratic mathematics education seeks to connect math class to these broader aims and claims that, in doing so, both mathematics class and the civic dimensions of schooling can be strengthened. There are a number of ways researchers have attempted to make the mathematics education–democracy link. Some attempt to use school mathematics as tools to help understand and analyze social inequities (Gutstein, 2006). Others start with acknowledgment of the ways school mathematics serves as a gatekeeper, allowing those who do well to move on to college and relegating those who do not to a noncollege track (Moses & Cobb, 2001). Still others work to undermine differential levels of attainment according to race, gender, or other category of marginalization (Boaler, 1997, 1998; Moses & Cobb, 2001; Curry, 2008; Villalobos, 2009).

Smith and I (Stemhagen & Smith, 2008) see each of these efforts as important and necessary, yet not entirely sufficient as means to bring about a democratic mathematics education. We argued that for mathematics class to be a site of democratic education, its content and aims need to be reconceptualized. We provided a blend of Dewey’s political philosophy with his philosophy of mathematics as the theoretical underpinning for our project. According to this framework, any attempts at democratic education must account for democratic societies’ requirements of internal cohesion and external interaction. Dewey explained: “[internal cohesion] signifies not only more numerous and varied points of shared common interest, but greater reliance upon the recognition of mutual interests . . . [external interaction] means not only freer interaction between social groups . . . but change in social habits” (1916, p. 86).

Smith and I extend this and write: “the very meaning and value of democracy is found in the development of individual capacity and the subsequent demand that citizens give back to society” (Stemhagen & Smith, 2008, p. 27). We use this as our foundation for what all schooling should foster and then consider how Dewey’s philosophy of mathematics can help make school mathematics reasonably fit into this wider democratic scheme. Dewey’s conceptualization of mathematics is humanistic and pragmatic; that is, he saw mathematics as a set of tools human have constructed to solve real problems in an ongoing effort to live better lives (McLellan & Dewey, 1900; Stemhagen, 2003). This way of thinking about mathematics, according to my work with Smith (2008), affords students the opportunity to engage in genuine problem solving and suggests that such efforts can help students recognize and develop their agency. Agency, here, means that students use their mathematical knowledge and skills to solve problems germane to their lives and even to make the world a better place.

**REFORM/DEMOCRATIC MATHEMATICS PEDAGOGY**

The reform orientation toward inquiry-based instruction, while not mandating the Deweyan democratic approach is certainly not inconsistent with it. While the definition of problem solving as mathematical activity, as set by the National Council of Teachers of Mathematics (NCTM), does not presuppose a particular solution method explicitly in order to foster agency, it can create the space for students to make choices about how best to go about solving particular problems. Thus, while not mandating democratic education, the NCTM approach does not put up barriers to it.
One inadequacy of problem solving as the standard for good teaching and learning is that nowhere does it state that the problems need to matter in anyone’s life beyond the mathematics class (see National Council of Teachers of Mathematics, 2000). Deweyan democratic mathematics education takes reform mathematics a step further—at least as it is articulated in Principles—by necessarily tethering the learning experiences of the individual to that of the group, thus helping to overcome the solitary aspects of most forms of mathematics education, which are major impediments to the cultivation of democratic principles in school mathematics. Thus, while there is certainly much overlap between reform and democratic mathematics, there are differences, too. Democratic mathematics proponents see themselves as working to forward many of the same interests as are other reformers; however they also see the cultivation of democratic ideals in mathematics class as an important way to make mathematics class better and to tie it into the broader goals of education and of social improvement (Stemhagen & Smith, 2008).

**STUDYING THE EFFECTS OF REFORM/DEMOCRATIC MATHEMATICS EDUCATION**

In mathematics education research, often the conclusion drawn is that there is a disconnect between teacher beliefs and practices—that is, teachers do not act in accordance with their beliefs (Alderton, 2008; Cooney, 1985). Leatham (2008) suggested that perhaps the question of whether particular teacher practices cohere with particular beliefs is the wrong question to ask and that researchers need to find ways to model more complex relationships than a simple linear one between a given belief and a given practice. Several researchers have taken Leatham’s challenge (Van der Sandt, 2007; Wilkins, 2008) and worked to add complexity to the sorts of phenomena that influence teacher practices. Such efforts have included teacher knowledge, pedagogical knowledge, beliefs about the nature of mathematics and teaching and learning, and issues related to teacher efficacy. Often these entities have been talked about as freestanding constructs, but it is also possible to focus on how these concepts manifest themselves in the systems of beliefs that teachers possess. Of particular note is that, while there is precedent in the psychological literature on efficacy (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996), in mathematics education research there has been a tendency to cordon off efficacy from other forms of belief.

Pajares wrote: “Teacher beliefs can and should become an important focus of educational inquiry but . . . this will require clear conceptualizations, careful examination of key assumptions, consistent understandings and adherence to precise meanings and proper assessment and investigation of specific belief constructs” (1992, p. 307). This study seeks to contribute to such an agenda. The section that follows attempts to clarify just what is meant by a “system of beliefs,” and a starting point for that is an assertion that the focus of this study is not conceptualizations or philosophies of mathematics in and of themselves but rather how it is that particular teachers appropriate, incorporate, and modify these conceptualizations in light of their other beliefs across a number of domains.

One strand of mathematics education research has been to consider what teachers believe in order to understand teacher behaviors (Clark, 1988; Cole, 1989; Fenstermacher, 1979; Nespor, 1987). Many beliefs have been considered in relationship to teacher practice, such as self-efficacy/self-concept (Pajares, 1992); subject-specific beliefs, such as the nature of mathematics (Ernest, 1989; Schoenfeld, 2001); the role of education—specifically mathematics education—in society (Dewey, 1916; Stemhagen & Smith, 2008); views of teaching and learning mathematics (Ball, 1991); beliefs and expectations of administrators (Cady, Meier, & Lubinski, 2006); and attitudes toward mathematics and the teaching of mathematics (Karp, 1991; Baumert et al., 2009).

Leatham (2006) argued that inconsistencies that have been identified between beliefs and practices (Alderton, 2008; Cooney, 1985; Herrington, Herrington, & Glazer, 2002) stem from a research paradigm that incorrectly assumes that teachers are able to articulate their beliefs and that there is a one-to-one correspondence between teacher utterances and researcher interpretation. Alternatively, Leatham’s theoretical framework situates teacher beliefs in a sensible system. This framework calls for a greater understanding of the complex interactions of various teacher beliefs and contextual contingencies. The sensible system framework requires the researcher to move beyond inquiring what teachers believe in order to investigate how the beliefs are arranged and interact as teachers make sense of their world and shape their actions.

**Development of Conceptual Model**

**BUILDING ON WILKINS’S MODEL**

Wilkins (2008), in response to Adler, Ball, Krainer, Lin, and Novatna’s (2005) call for larger scale quantitative studies to look at teachers’ knowledge, beliefs, attitudes, and practice, sought to model the relationship between these variables in elementary teachers. Wilkins found beliefs to have the strongest relationship to teacher practices and also that they served a mediating role for knowledge and attitudes related to mathematics. Content knowledge was negatively related to inquiry-based instructional practices. This finding was consistent with literature suggesting that teachers with strong content knowledge in mathematics may be more likely to use traditional teaching methods, rules, and procedures (Mewborn, 2001).

Wilkins’s model positions teacher background characteristics as coming temporally prior to content knowledge and attitudes. Content and attitudes, next, come prior to instructional beliefs, all of which come prior to instructional practice (see Figure 1). For the sake of this study, I take as given that experience—via teacher background—comes temporally prior to relevant knowledge and beliefs, although emerging beliefs could be thought of as shaping future experience. I also agree that practice is influenced by everything prior in Wilkins’s model.

In terms of departures from Wilkins’s model, I first blur the distinction between beliefs and attitudes, as there has been some conceptual confusion between the two throughout this vein of research (Pajares, 1992). Also, attitudes can be conceptualized as certain kinds of beliefs, but are beliefs nonetheless (Bandura et al., 1996). Since how different kinds of beliefs relate to each other in an
overarching system of beliefs is the object of study, in this model they are merely separate instances of belief constructs (see Figure 2).

The blurring of the belief-attitudes distinction serves a second purpose. Contra to Wilkins’s model, content knowledge does not come prior to beliefs. Instead, it is posited that beliefs about teaching mathematics can and should influence attitudes about mathematics and possibly even content knowledge (Kitcher, 1983). Hence, beliefs-attitudes are understood as important influencers of practice. I do not deny the influence of content knowledge on practice, and this project starts with Wilkins’s findings that teacher beliefs most strongly relate to teacher practice.

I do not envision practice as the end of the teaching act. Instead, practice—at least robustly reflective practice—informs beliefs-attitudes and, ultimately, future practices. Thus, an arrow is added from practices back to the web of beliefs to symbolize the circular nature of such interactions. There is a venerable history of nuanced models adding such facets. See for example Dewey’s (1896) additions to the more simplistic behaviorist stimulus-response model.

A Sensible Web-of-Beliefs Model
A central tenet of this project is that beliefs do not relate to practices in the absence of other beliefs, and this model seeks to account for the interrelationships between beliefs that ultimately affect practice. Thus, this model features a set of belief constructs that exist in correlation to one another. This set of correlations is crucial to understanding teacher practice. So, particular teacher practices are considered in light of the entire system or web of beliefs (see Figure 3).

I have conceptualized this model to begin to study the ways systems of teacher belief affect practice. I see teacher beliefs in a number of arenas as relevant to teacher practice and this list of belief constructs is decidedly nonexhaustive, somewhat exploratory, and selected as much because these are beliefs that interested me as for any other reason. In essence, this provides a place to start this sort of work. Also, I have attempted to capture this situation with a category called perceived influences, and for the sake of this model I see its place as a potential mediator between the sensible web of beliefs and practice. For example, regardless of a teacher’s beliefs, it is possible that a teacher might act in a certain way because of what she sees as pressure from her school administration to teach a certain way.

Turner (2007) argued that social-science research models should be as simple as possible, but no simpler, and he argued that current educational research models, if they are to inform policy and practice, need to be more complex than they currently are. In this spirit, this model is designed to be sufficiently complex to begin to test the idea that mathematics teachers’ beliefs are always in relation to one another but conceptually simple enough to still be useful as a model. In sum, this model posits teacher beliefs as existing in a constellation and considers how the whole constellation of beliefs relates to teacher practices. I started with Wilkins’s (2008) conceptual model and modified it to accommodate and focus on a beliefs web. Furthermore, I included the possibility of practice being affected by external forces (see Figure 3). Teachers’ content knowledge was specifically not included in order to focus on teacher beliefs and to maximize the model’s ability to illuminate belief-practice relationships. A brief description of how different parts of the model are operationalized is included in the methods section.

Research Questions
This study seeks to investigate the relationships between teacher backgrounds, their webs of belief, perceived external influences, and teacher practices. The following questions served to focus the inquiry:
1) Are there tendencies for beliefs to cluster together for certain groups of teachers? How do these belief types relate to teacher background and practice?
2) Does the web of beliefs model provide continuity between teacher beliefs and practices?
3) Are there individuals for whom beliefs and practices still seem to be at odds?
4) Are other beliefs or influences moderating the sensible system in these situations?
5) Is democratic mathematics education a tenable theoretical construct and if so, how does it relate to other beliefs and practices?

For the sake of this article, question 5 is foregrounded. That said, here I am not solely concerned with the research/methodological dimensions of the construct—I am also concerned with making claims about how democratic mathematics education did...
or did not relate to the work of the teachers in this study. As a result, questions 1-4 should not be ignored, as they provide important tools of analysis to help us grapple with the democratic question.

**Method and Design**

**PROCEDURES**

**Participants.** This study involved a nonexperimental design that employed online teacher surveys as the primary data-collection method. The teacher surveys were administered to mathematics educators (grades four through eight) in seven local school districts during the spring of 2009. These districts represent rural, suburban, and urban populations from a state in the southeastern United States. There were a total of 323 participating teachers. Once incomplete, and hence unusable, surveys were discarded, the final \( n=249 \).

**Instrument and instrument pilot.** Most items for the survey used in this study were modified from existing studies in order to specifically address mathematics. Belief constructs for which no adequate surveys were found (e.g., democratic math education) were developed from existing, peer-reviewed theoretical literature. After constructing the instrument, items related to each construct were sent to seven content experts to sort and critique (e.g., whether the stated practice was transmittal, constructivist, or democratic). The revised instrument was then piloted to gather reliability and validity information and further ensure the quality of the instrumentation. A pilot study \( (n=27) \) involving experienced mathematics teachers provided quantitative and qualitative data and both were used to improve the instrument.

**Quantitative measures**

The final survey collected information on teacher background/demographic variables, teacher beliefs, self-reported teacher practices, and perceived influences. Each is briefly described below:

**Teacher background.** Twelve teacher background/demographic variables were considered in this study: number of college-level math courses taken, highest degree attained, content area of highest degree, grade level taught, years of teaching experience, perception of socioeconomic level of students taught, whether categorized as highly qualified, whether provisionally licensed, whether special education teacher, whether completed math coach program (regardless of whether currently a math coach), whether the teacher has access to a math coach/specialist, and gender. Each of these demographic variables was measured using a single question with forced-choice response categories.

**Web-of-belief constructs.** This component was the most complex, as it attempted to capture the dynamic ways teachers' particular beliefs relate to their other beliefs. For the sake of this study, I settled on six particular belief constructs: constructivist beliefs about the nature of mathematics, absolutist beliefs about the nature of mathematics, constructivist pedagogical beliefs, transmittal pedagogical beliefs, democratic pedagogical beliefs, and beliefs about mathematics teaching efficacy. Subscales for each of these constructs used 5-point Likert-type scale questions. While several of the constructs seem diametrically opposed (e.g., absolutist versus constructivist philosophies and constructivist versus transmittal teaching beliefs), this web model is designed to test the idea that individuals often hold somewhat seemingly contradictory ideas in tension with one another. See the conceptual model (Figure 3) and findings section for background and discussion of these phenomena, respectively.

The two constructs related to the nature of mathematics, constructivist and absolutist, were designed to examine each teacher’s philosophy of mathematics, be it implicit or explicit (see the literature review for further description of these constructs). The items for these constructs were modified from Baumert et al. (2009) and Ernest (2006). Their internal consistency was measured with Cronbach’s Alpha—constructivist beliefs about the nature of mathematics’ Alpha was .614 and absolutist beliefs about the nature of mathematics’ Alpha was .698 (see Table 1 for Cronbach’s Alpha scores for all constructs).

The belief constructs related to pedagogy refer to the orientation teachers have toward teaching and learning. Constructivism
refers to whether the teaching-learning enterprise is primarily about student creation and construction, while the transmittal construct measures the degree to which teachers see teaching and learning as primarily about the transmission of knowledge and skills from teacher to students. The items in these sections were modified from Wilkins (2008), Baumert et al. (2009), and Ernest (2006). I started with separate constructs for beliefs about the teaching of mathematics and the teaching of other subjects. Following the literature, I hypothesized that teachers would have different beliefs about mathematics teaching and teaching other content. As it turned out, this distinction could not be confirmed, and the collapsing of the categories led to constructs with strong reliability scores, .757 for constructivist pedagogy beliefs and .830 for transmittal.

The democratic pedagogy beliefs construct considered the ways in which teachers conceptualize the role of teaching and learning given wider democratic sociopolitical spheres. For the sake of this project, I measured the degree to which teachers view the role of education and mathematics as means to democratic ends. The items for this section came from the theoretical literature, particularly Stemhagen (2009) and Stemhagen and Smith (2008). This construct was measured by binary, forced-choice questions. Its Cronbach’s Alpha of .837 suggests an acceptable internal consistency.

The final belief construct was mathematics and mathematics teaching self-efficacy. A modified version of Wilkins’s Likert-type scale instrument was designed to measure mathematics teachers’ liking of mathematics, feelings of success with mathematics, enjoyment while or liking of teaching mathematics, and feelings of success as a teacher of mathematics. The Cronbach’s Alpha for the efficacy construct was .704.

Teacher practices. Wilkins (2008) studied the relationship between beliefs about inquiry-based instruction and inquiry-based instructional practices. In this model, the sensible web-of-beliefs contains belief constructs that vary along transmittal and constructivist orientations as seen in Baumert et al. (2009) as well as belief constructs that vary from autocratic to democratic orientations (Stemhagen & Smith, 2008). The added complexity in the conceptualization of teacher belief systems necessitated added complexity in the measure of reported practice. Therefore, in this study, the construct teacher practices was designed to be sensitive to practices ranging from transmittal-autocratic to constructivist and democratic.

In order to measure these differences in reported practices, teacher-practice subscales were tested in the areas of constructivist, democratic, and autocratic-transmittal practices. Questions for these subscales employed items from Wilkins (2008) and modified items from Baumert et al. (2009), as well as questions drawn from the theoretical literature. These questions used a 5-point Likert-type scale ranging from never to frequently, regarding how often teachers engage in certain practices. The Cronbach’s Alphas for transmissive, constructivist, and democratic practices were .680, .600, and .662, respectively.

Perceived influences. The above constructs and corresponding survey subscales were intended to provide data for research questions 1–3 and 5. Question 4 addresses the possibility that the limited beliefs included within this sensible belief web still appear to be at odds with teacher practices. In the event that such cases exist, this study employed an additional subscale. Participants were asked to rank the top five external influences from a list of nine choices: state/federal-mandated student performance standards (e.g., SOL, NCLB), central-district office policy (pacing guides, etc.), school administrators, professional development, math specialists/coaches, parents of students, availability of resources (materials, money, etc.), colleagues’ practices, and other.

Factor analysis of the teacher survey suggested the presence of the aforementioned subscales. As described above, construct reliability testing and further factor analysis were employed to compute a measure of effect size.

MODES OF DATA ANALYSIS
A focus of this project is the consideration of the relationships between teacher belief constructs within their sensible web-of-beliefs. Comparing belief constructs and the overall web-of-beliefs to reported practice is an essential if not novel contribution in this area of research. This analysis also looks more specifically at tendencies of belief for different teacher subgroups. Finally, the analysis also considers other factors identified as perceived influences that might mediate the relationship.

In terms of specific methods of analysis, the employment of a correlation matrix was the initial means to consider how the various belief constructs tended to relate to each other. Next, I looked at descriptive statistics related to demographic and background questions and examined whether these patterns held true for different subgroups within the population. One-way analysis of variance was the primary technique employed during this phase of the analysis. Additionally, Cohen’s d was computed to provide a measure of effect size.

Structural equation modeling (SEM) was employed to test the conceptual model. The relationship between individual belief

<table>
<thead>
<tr>
<th>Belief/Practice Constructs</th>
<th>Cronbach’s Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Teaching Self-Efficacy</td>
<td>.704</td>
</tr>
<tr>
<td>Absolutist Nature of Mathematics</td>
<td>.698</td>
</tr>
<tr>
<td>Constructivist Nature of Math</td>
<td>.614</td>
</tr>
<tr>
<td>Transmissive Teaching</td>
<td>.830</td>
</tr>
<tr>
<td>Constructivist Teaching</td>
<td>.757</td>
</tr>
<tr>
<td>Democratic Teaching</td>
<td>.837</td>
</tr>
<tr>
<td>Transmissive Practice</td>
<td>.680</td>
</tr>
<tr>
<td>Constructivist Practice</td>
<td>.600</td>
</tr>
<tr>
<td>Democratic Practice</td>
<td>.662</td>
</tr>
</tbody>
</table>
constructs and an overall web-of-beliefs was considered, as was the web to individual types of practice and to an overall latent practice variable. The intention was to build the perceived influences into the SEM. However, because two influences represented an overwhelming percentage of response, there was no way to meaningfully incorporate this set of factors into this portion of the analysis. The results of the perceived-influences questions, however, are interesting and relevant in their own right and are discussed in the findings section.

**Findings/Results**

In all, 323 teachers responded to the online survey (a response rate of approximately 42%). Once unusable surveys were filtered, the working data set had \( n=249 \). The response rate is approximate, as school districts disseminated the survey’s website address and not all participating schools districts provided the researchers with the number of teachers who received the link.

**SELECTED DEMOGRAPHIC CHARACTERISTICS**

There are some demographic findings particularly worthy of note. The late-elementary (grades four through five) and middle school (grades six through eight) split was approximately 62% to 37%. Approximately 12% of fourth-through-eighth-grade math teachers surveyed reported having taken zero to two college-level math classes and 53% have taken five or less. Over half of the respondents had a master’s degree (54.7%). Only 10% of the respondents reported that their highest degree is in mathematics, and the vast majority’s highest degree was in teaching or education. Level of experience is well distributed, with approximately half of the respondents reporting less than ten years in the classroom and half reporting ten or more years in the classroom. Approximately 17% were not categorized as highly qualified, and 9% percent were provisionally licensed. Fourteen percent of the respondents reported having undergone math-specialist training, and 82% had access to a math coach/specialist. Finally, special education teachers made up 17% of the respondents, and they were roughly equally divided between those in inclusion versus self-contained environments.

**CORRELATIONS**

This study found a number of significant correlations between constructs. Since teacher webs-of-belief are a point of focus and since the relationship between the overall web and teacher reported practice is a second area of focus, the correlation matrix has been broken into two tables: beliefs and beliefs-to-practice (Tables 2 and 3, respectively). While the belief matrix does provide an initial means to explore the web-of-beliefs, it only considers the relationship between each individual belief and other beliefs. It does not, in any robust way, represent the way webs of belief (as I have described them) operate. Likewise, Table 2’s description of the beliefs-to-practices correlations only examines the relationship between each belief construct and each type of reported practice. The structural equation modeling that was undertaken and is reported below is an initial foray into representing the complexity of the web-of-beliefs model. The possible meanings of these relationships are considered later in this article.

**BELief AND Practice DIFFERENCES BETWEEN SUBGROUPS**

Analysis of variance (ANOVA) was performed as a means to identify tendencies toward differences in response according to subgroupings. In cases where the ANOVA identified significant differences, T-tests were run and Cohen’s \( d \) effect size was computed to present a standard metric of differences between groups.

Effect size was calculated for differences in beliefs according to teachers in grades four through five versus those in grades seven through eight (see Table 4). Sixth grade was left out of the analysis because it seemed to obscure differences between late-elementary and middle school teacher responses. This was justifiable because the sixth-grade subgroup did not show significant differences between either of the other two grade groups. While significant in many areas, the effect size was only relatively large in a few areas (see Table 4). Elementary school teachers were much more likely as a group not to possess an absolutist philosophy of mathematics.

<table>
<thead>
<tr>
<th>Table 2. Belief Construct Correlation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature of Math (Absolutist)</td>
</tr>
<tr>
<td>Nature of Math (Constructivist)</td>
</tr>
<tr>
<td>Transmittal Pedagogy</td>
</tr>
<tr>
<td>Constructivist Pedagogy</td>
</tr>
<tr>
<td>Democratic Pedagogy</td>
</tr>
<tr>
<td>Mathematics Teaching Efficacy</td>
</tr>
</tbody>
</table>
They were also less likely to hold traditional or transmissive beliefs about teaching and also less likely to engage in transmissive teaching practices. Conversely, as a group, they were much more likely to possess constructivist beliefs about teaching.

The data were also parsed according to teachers who have undergone specialist training and those who have not (see Table 5). Significant differences were found in several categories and the effect size was reasonably large in several areas. Teachers who have undergone mathematics-specialist training were more likely to possess a constructivist philosophy of mathematics, a constructivist pedagogical outlook, and higher self-efficacy with regard to teaching mathematics. They were less likely to hold a transmittal outlook toward pedagogy.

**TESTING THE CONCEPTUAL MODEL**

The development of a conceptual model that does not assume a singular, linear relationship between a belief and a given teacher’s reported practice is an important component of this study. Building on this model, this study identifies the relationship between certain belief constructs, a latent web-of-beliefs, and reported teacher practice. Additionally, this study sought to explore potential mediating factors/perceived influences between beliefs and practices. These perceived influences were measured but there was a limited response range with a large majority of respondents selecting two response categories (see Table 6). Though providing interesting data, the restricted range of response made it necessary to exclude perceived influences from the testing of the conceptual model.

Structural-equation modeling allowed quantitative consideration of the relationships between the various belief constructs and an overarching system of belief that is positively related to practice. There are indications that the SEM supports the conceptual model. Two metrics—comparative fit index (.919) and root mean square effort of approximation (.10)—are both within reasonable fit margins (Kim, 2006). Unfortunately, these versions of the model did not lead to reportable estimated correlations, and hence all that can be concluded at this time is that the particular model is not viable. More in-depth employment of SEM needs to be undertaken prior to making any claims about the statistical support for this model.

**Table 3. Significant Relationships Between Belief Constructs and Practice**

<table>
<thead>
<tr>
<th>Belief Constructs</th>
<th>Transmissive Practices</th>
<th>Constructivist Practices</th>
<th>Democratic Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutist Nature of Math</td>
<td>.365</td>
<td>.360</td>
<td>.300</td>
</tr>
<tr>
<td>Constructivist Nature of Math</td>
<td>-.172</td>
<td>.360</td>
<td>.000</td>
</tr>
<tr>
<td>Transmittal Pedagogy</td>
<td>.352</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>Constructivist Pedagogy</td>
<td>-.229</td>
<td>.169</td>
<td>.168</td>
</tr>
<tr>
<td>Math Teaching Self-Efficacy</td>
<td>.000</td>
<td>.010</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4. Comparison of Fourth-and-Fifth-Grade and Seventh-and-Eighth-Grade Teachers**

<table>
<thead>
<tr>
<th>Construct</th>
<th>Cohen's $d$</th>
<th>Sig.</th>
<th>4th–5th Mean</th>
<th>7th–8th Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutist Nature of Mathematics</td>
<td>-0.33</td>
<td>.01</td>
<td>3.1</td>
<td>3.46</td>
</tr>
<tr>
<td>Mathematics Teaching Self-Efficacy</td>
<td>-0.25</td>
<td>.01</td>
<td>4.3</td>
<td>4.52</td>
</tr>
<tr>
<td>Constructivist Practices</td>
<td>0.34</td>
<td>.00</td>
<td>3.44</td>
<td>3.12</td>
</tr>
<tr>
<td>Constructivist Pedagogy</td>
<td>0.23</td>
<td>.016</td>
<td>4.13</td>
<td>3.92</td>
</tr>
<tr>
<td>Transmittal Pedagogy</td>
<td>-0.33</td>
<td>.003</td>
<td>2.68</td>
<td>3.03</td>
</tr>
</tbody>
</table>

**Table 5. Comparison of Specialist Training and No Specialist Training Groups**

<table>
<thead>
<tr>
<th>Construct</th>
<th>Cohen's $d$</th>
<th>Sig.</th>
<th>Specialist Training</th>
<th>No Specialist Training</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constructivist Nature of Mathematics</td>
<td>0.50</td>
<td>.000</td>
<td>4.43</td>
<td>3.97</td>
</tr>
<tr>
<td>Transmittal Pedagogy</td>
<td>-0.37</td>
<td>.015</td>
<td>2.4</td>
<td>2.2</td>
</tr>
<tr>
<td>Constructivist Pedagogy</td>
<td>0.39</td>
<td>.001</td>
<td>4.39</td>
<td>4.03</td>
</tr>
<tr>
<td>Mathematics Teaching Self-Efficacy</td>
<td>0.38</td>
<td>.003</td>
<td>4.63</td>
<td>4.32</td>
</tr>
</tbody>
</table>
discuss}

**Table 6. Perceptions of External Influences**

<table>
<thead>
<tr>
<th>External Influence</th>
<th>Selection Frequency</th>
<th>Most Frequent Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>State/Federal-Mandated Student Performance Standards (e.g., SOL, NCLB)</td>
<td>90%</td>
<td>1</td>
</tr>
<tr>
<td>Central-District Office Policy (Pacing Guides, etc.)</td>
<td>84%</td>
<td>2</td>
</tr>
<tr>
<td>Availability of Resources (Materials, Money, etc.)</td>
<td>62%</td>
<td>4</td>
</tr>
<tr>
<td>School Administrators</td>
<td>60%</td>
<td>3</td>
</tr>
<tr>
<td>Professional Development</td>
<td>58%</td>
<td>4</td>
</tr>
</tbody>
</table>

**Discussion**

**SINcigNificance**

The single most obvious and important outcome of this study is support for the idea that beliefs do, in fact, seem to matter. The initial correlation studies showing the strongest relationship between teacher practice and underlying philosophy (as opposed to between teacher practice and beliefs about teaching) is certainly an interesting finding, particularly in light of the fact that there is so little explicit attention to philosophy of mathematics in both mathematics and mathematics education programs (Thompson, 1992). This finding corroborates findings from other studies as to the importance of beliefs and philosophies (e.g., Raymond, 1997; Ernest, 1989; Wilkins, 2008).

I am encouraged by the high number of significant relationships between constructs and the fact that the directions and often the strength of the relationships were consistent with what was hypothesized. Philosophical beliefs tended to correlate with both pedagogical beliefs and reported practice (see Figures 4 and 5). While not particularly surprising, these findings bear consideration for teachers, teacher educators, and others involved in professional development. Teachers’ philosophies of mathematics—that is, how they view the nature of their subject matter—are highly correlated with their pedagogical orientation. Philosophy of mathematics is also relatively highly correlated with reported practices. Teachers’ pedagogical orientation is also correlated with practices, although the strength of these relationships is, for the most part, slightly weaker than the others under scrutiny. Any of these relationships could be cited as evidence supporting the idea that teachers’ beliefs (philosophical and/or pedagogical) should not be ignored. In essence, this study suggests that if one wants to change a teachers’ practice, say from transmittal to constructivist, one should not merely teach new practices, one needs to provide opportunities for the teacher to reconsider other relevant beliefs—e.g., general beliefs about teaching and learning, ideas about the nature of mathematics, and very possibly other beliefs that were not under scrutiny in this study.

Whether increased attention to teacher beliefs leads to increased achievement on standardized tests is one question and whether studies such as this one ought to try to make such links is another. The latter question is considered in this article’s final section. Regarding the former, teachers’ classroom practices have been linked to student achievement (e.g., Levybusce & Zupancic, 2009; Wallace, 2009). Furthermore, it does not seem a stretch to claim that, even if student performance is the only desired end, focus on teacher beliefs is warranted.

Figures 4 and 5 also depict a correlation disparity between transmittal pedagogy to practice and constructivist pedagogy to practice (r = .35 and r = .17, respectively). One interpretation of this data, particularly when coupled with the perceived external influence data (Table 6), is that teachers find it more difficult to enact constructivist beliefs in the classroom than they do transmittal ones. In other words, turning beliefs into corresponding practices, given NCLB, SOL, and district-level policy appears to be a less complicated enterprise if one possesses a traditional sense of what ought to be happening in the classroom.

That acting on constructivist beliefs about mathematics and its teaching and learning appears more difficult than does acting on traditional, transmittal ones is particularly interesting (and potentially disturbing) given the fact that the NCTM, arguably the most important mathematics education organization, is clear and unequivocal in their endorsement of constructivist teaching methods (National Council of Teachers of Mathematics, 2000).

**Summary: Revisiting the Initial Research Questions**

To wrap up this section, let us now reconsider the initial research questions. The first question addressed whether teachers’ web-of-belief tend to cluster together for certain groups of teachers and how these belief types relate to teacher background and practice. The effect size differences between various demographic subgroups sought to engage this particular inquiry. Two comparisons yielded the highest number of significant differences. Upper elementary teachers (fourth through fifth grades) and middle school teachers (seventh through eighth grades) differed in terms of philosophy, pedagogical orientation, practice, and efficacy levels. Teachers who had undergone math-specialist training were much more likely to possess constructivist philosophical and pedagogical orientations, more likely to score high on the efficacy measure, and much less likely to possess a transmittal pedagogical orientation than their peers who had not undergone such training.

Regarding the question of whether the web-of-beliefs model provides continuity between teacher beliefs and practices (question 2), this work suggests that the web model can provide this continuity. Although both beliefs about teaching and about the nature of mathematics show statistically significant relationships with constructivist teaching practices, it is the relationship between a constructivist philosophy of mathematics and constructivist teaching practices that was the strongest (r = .36). Likewise, a teacher’s transmissive teaching practices were roughly as likely to be informed by her philosophy of mathematics as her beliefs about teaching (r = .37 and .35, respectively). Furthermore, structural
equation modeling, while inconclusive, still seems to provide a promising technique to model web-of-belief-practice relationships.

As to whether beliefs and practices still seem at odds and whether other beliefs-influences are moderating the sensible system in these situations (questions 3 and 4), often the relationships between beliefs and corresponding practices seemed reasonably strong. As noted above, this was particularly true with transmittal-absolutist beliefs and transmittal practices and less so with constructivist constrasts.

There were clear patterns regarding teachers’ perceptions about external influences to their practice (see Table 6). It also bears mentioning that while pedagogical orientations (constructivist and transmittal) showed reasonably high correlations with their counterparts in practice, they were both much more highly correlated with their corresponding philosophical outlook (constructivist or absolutist). These findings support the idea that the relationship between beliefs and practices is not best understood as existing in simple one-to-one dyads. Further quantitative and qualitative study could shed light on individual teachers who show strong tendency toward certain clusters of philosophical and pedagogical beliefs (e.g., constructivist philosophical and pedagogical orientations) and who tend toward seemingly disconnected practices (transmittal, in the case of the example above).

The last research question asked whether democratic mathematics education is a tenable construct. There is much to suggest that it is. In expert review, participants had no trouble differentiating democratic beliefs and practices from others. At .837, democratic pedagogy had the highest Cronbach’s Alpha of all constructs (see Table 1). This measure provides evidence that the subscale is internally consistent. While these are reasons for optimism, it should be noted that, in this study, there was considerable overlap between constructivist and democratic teaching practices ($r = .598, p < .000$). This is not completely unexpected as, theoretically, teacher intent is an important difference between constructivist and democratic orientations and the practice constructs do not lend themselves to consideration of intent. Further supporting this theoretical position, the corresponding belief constructs that do address intent, constructivist pedagogy and democratic pedagogy, have a much lower relationship ($r = .308$). Simply put, there is question as to the practical differences between teaching for democracy and constructivist teaching. For example, an activity designed to foster democratic agency might not look all that different from one designed to facilitate student construction of math knowledge (at least one strain of constructivist pedagogy). Both could place student agency at the center of the activity. The primary difference between the two is the aim of the activity. The lower correlation between visions of constructivist and democratic pedagogy (as opposed to reported practice) does suggest that this study was subtle enough to be able to note this issue of intent. That said, much work in teasing out the differences between the two remains to be done if democratic mathematics education is to thrive as a freestanding philosophy-practice of mathematics education.
Stepping Back from the Study: Empirical Research and the Democratic Aims of School

As to the meta-level issue of carrying out empirical research projects related to the broad aims of schooling, particularly democratic ones: There were tensions as this empirical project moved through its various stages. Negotiating and reflecting on these tensions is worthwhile, as through this process I, a philosopher of education, came to better understand the perspective of other public school stakeholders. Likewise, it is my hope that through participation in this project my voice—an alternative voice in the context of PK–12 practitioner-administrator discourse—was heard and perhaps even appreciated. What follows is a very brief presentation of some of the resistance and tension experienced at the various stages of the project.

In the initial meetings with the study team, questions were raised about whether the MERC board would approve a study not immediately focused on test scores. Fairly quickly for some study team members, this gave way to a realization that we could make a study that might help them with their concerns, namely how to contend with teachers who, after having been taught (usually in in-service professional development events) to teach in a constructivist manner, tend to revert to more traditional teaching modes. One building-level math specialist asked, “Will this help us understand why we teach them to be constructivists and they don’t practice that way?” Additionally, there was concern that there isn’t a strong enough link in the literature between reported practice and test scores. Their reason for this concern was that the group felt that such a link would be the kind of support that would make their superiors more comfortable with the study.

One interesting and somewhat disappointing strain of commentary was the expression of consternation that teachers were being introduced to ideas (the democratic education questions) that would somehow be damaging or distracting from their jobs. One study-team member wondered whether we would disturb/upset the teachers with such aims-related and politically oriented material. Perhaps even more troubling, at the time of the survey dissemination, one school-district central office employee, after reading the survey, asked why we needed to ask such questions during this time of year when teachers were busy with the requirements of year-end testing.

At both meetings with the policy and planning board, there was resistance to a study that was not immediately and clearly linked to test-score improvement. Indeed, in preparation for the initial meeting, I purposefully highlighted the pedagogical questions on constructivist and transmittal teaching methods. Still, the first meeting was a rough and sometimes acrimonious one in which I at times felt like a country lawyer addressing the Supreme Court. The final meeting where I presented my findings went a little which I at times felt like a country lawyer addressing the Supreme Court. The final meeting where I presented my findings went a little better.

At this point, I explained that I felt the need to go on record that I saw it as incredibly unfortunate that we had to have this conversation and that test scores are not the ultimate measure of good teaching and learning. That said, I also acknowledged that the context within which everyone exists drives these concerns about test-score improvement. After a pause, a superintendent asked whether I could add any information about links between beliefs and student performance, adding something along these lines: “We use MERC as a way to justify good teaching—can you put something in the report that links beliefs to empirical outcomes so that we can justify acting on the recommendations?”

I did get a sense that, if nothing else, I had gained some perspective about how well-meaning superintendents and other administrators negotiate the test-score mania that has engulfed public schooling. Jesse Senechal, a graduate student and MERC fellow who attended the meeting, aptly summed up what went on. His comments highlight one important role that democratically oriented empirical work can play in the quest to improve schooling and how this type of project, in spite (or perhaps because) of its tensions is important:

I think what stood out was the one district representative who questioned the relevance of the democratic teaching model… I don’t remember exactly what he said, but I think his point was that democratic teaching didn’t seem to have much relation to student achievement (within his framework, test scores, the bottom line). And he was right. I’m glad he spoke up about it. It gave you an opportunity to talk about the elephant in the room. It led to awkward silence and several attempts by various folks to make sense of the fact that teaching for democracy and teaching for testing are antithetical enterprises. It exposed some truth… I’m fairly certain that everyone in that room knows that our test fetish is misguided. That there is this general silence on the issue (brought about by the political interests that support testing) is the problem.

What this episode made me think about in the days that followed was the benefit of this disruption. It also made me think that we need to develop substantial arguments that continue this disruption. I think Labaree’s framework (1997) is useful in this respect. What it suggests is that the goal of democratic equality (while not currently preeminent) nonetheless resides at the foundation of our collective (historical) understanding of what schools should be. (Just look at the rhetoric we employ—no child left behind). It’s not a stretch to think that this understanding could be reawakened as more than rhetoric. The absurdity of the silence around our educational purpose is ready for a challenge. (J. Senechal, personal communications, October 28, 2009).

References


